

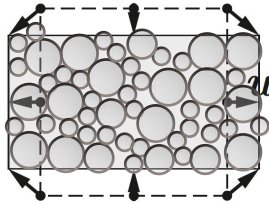
CIVIL-408

Multiscale Modeling in Mechanics

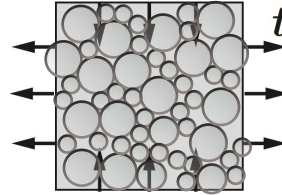
Prof. Kostas Karapiperis

Exercises - Week 7

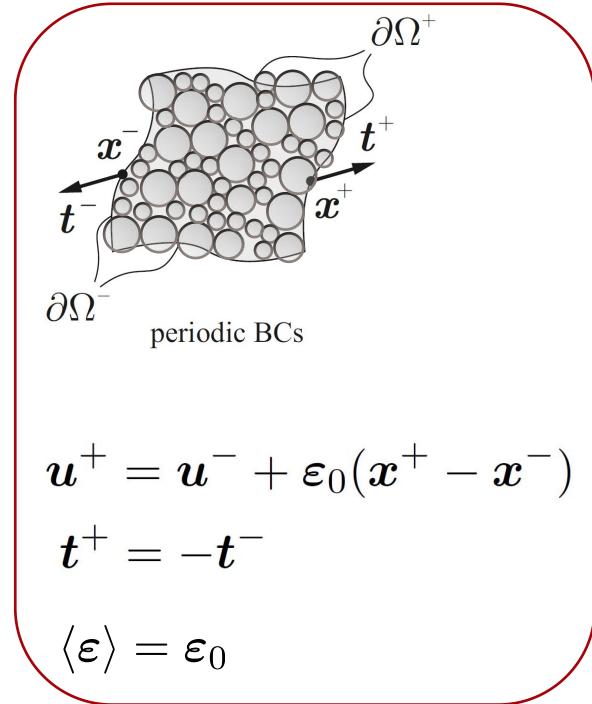
Different types of boundary conditions



affine displacement BCs



uniform traction BCs



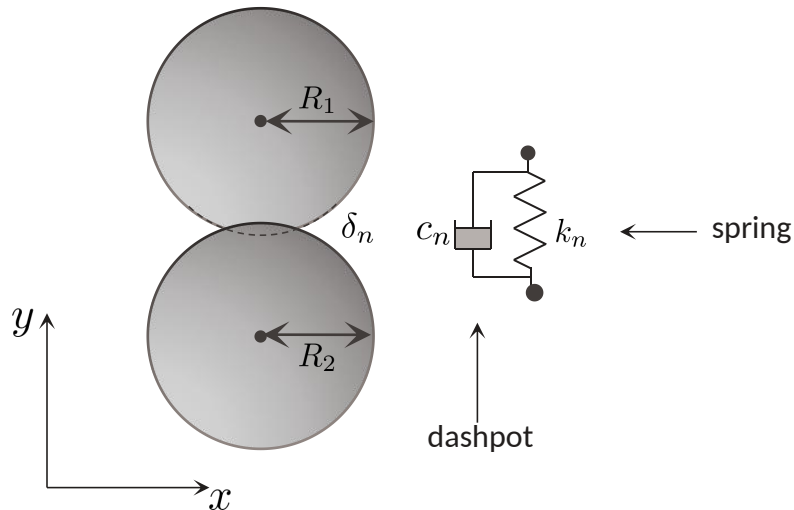
Linearized kin.: $\mathbf{u} = \boldsymbol{\varepsilon}_0 \mathbf{x}$

$$\mathbf{t} = \boldsymbol{\sigma}_0 \mathbf{n}$$

RVE averages: $\langle \boldsymbol{\varepsilon} \rangle = \boldsymbol{\varepsilon}_0$

$$\langle \boldsymbol{\sigma} \rangle = \boldsymbol{\sigma}_0$$

Particle interaction



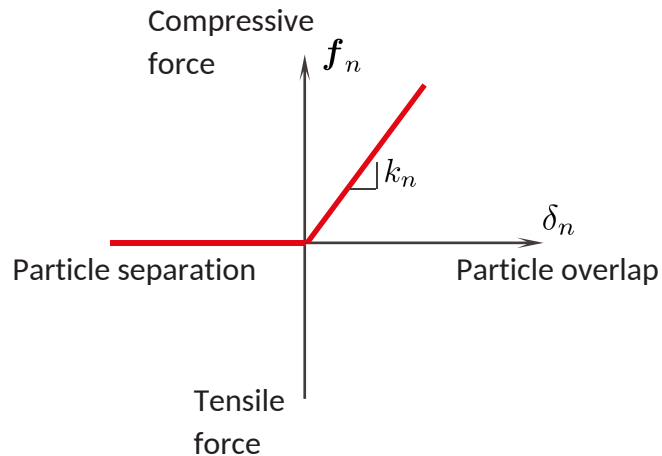
Overlap:

$$\delta_n = R_1 + R_2 - \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Contact normal:

$$\mathbf{n} = \frac{\mathbf{x}_1 - \mathbf{x}_2}{\|\mathbf{x}_1 - \mathbf{x}_2\|}$$

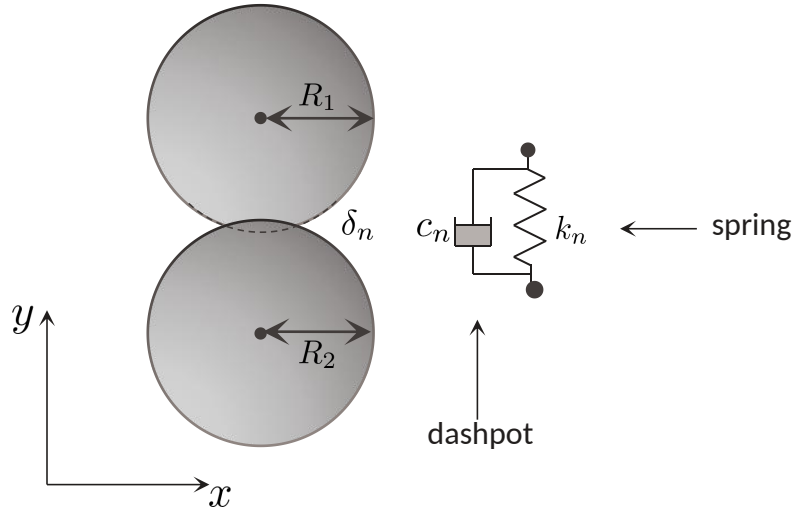
Hookean contact stiffness



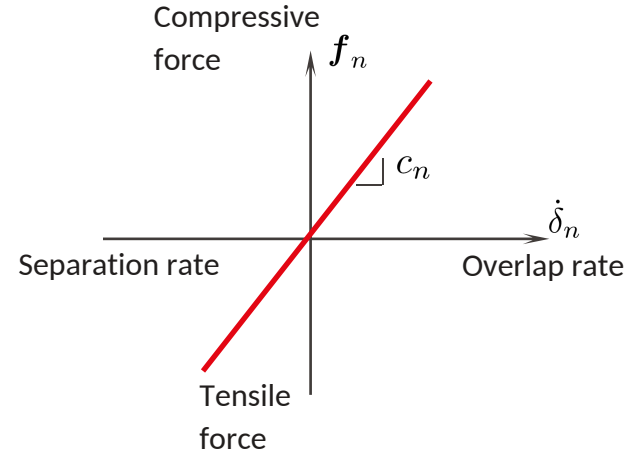
$$f_n = k_n \delta_n$$

$$k_n = \text{const}$$

Particle interaction



Viscous energy dissipation due to restitution



Overlap:

$$\delta_n = R_1 + R_2 - \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Contact normal:

$$\mathbf{n} = \frac{\mathbf{x}_1 - \mathbf{x}_2}{\|\mathbf{x}_1 - \mathbf{x}_2\|}$$

$$f_n = k_n \delta_n + c_n \dot{\delta}_n$$

$$c_n = -2 \frac{\ln e}{\sqrt{\pi^2 + \ln e^2}} \sqrt{m^* k_n}$$

$$m^* = \frac{m_1 m_2}{m_1 + m_2}$$

The **critical time step** in DEM is mainly determined by:

- a) the mass and stiffness of the particles
- b) the simulation domain size
- c) the number of particles
- d) the density of the packing only

The **Verlet radius**:

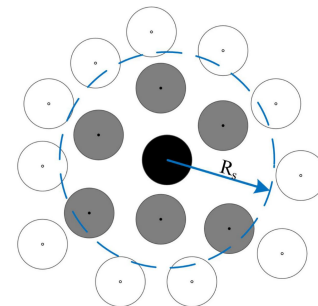
- a) determines the size of the RVE
- b) defines the area around each particle where contact is checked
- c) is the maximum particle radius in the system

The **critical time step** in DEM is mainly determined by:

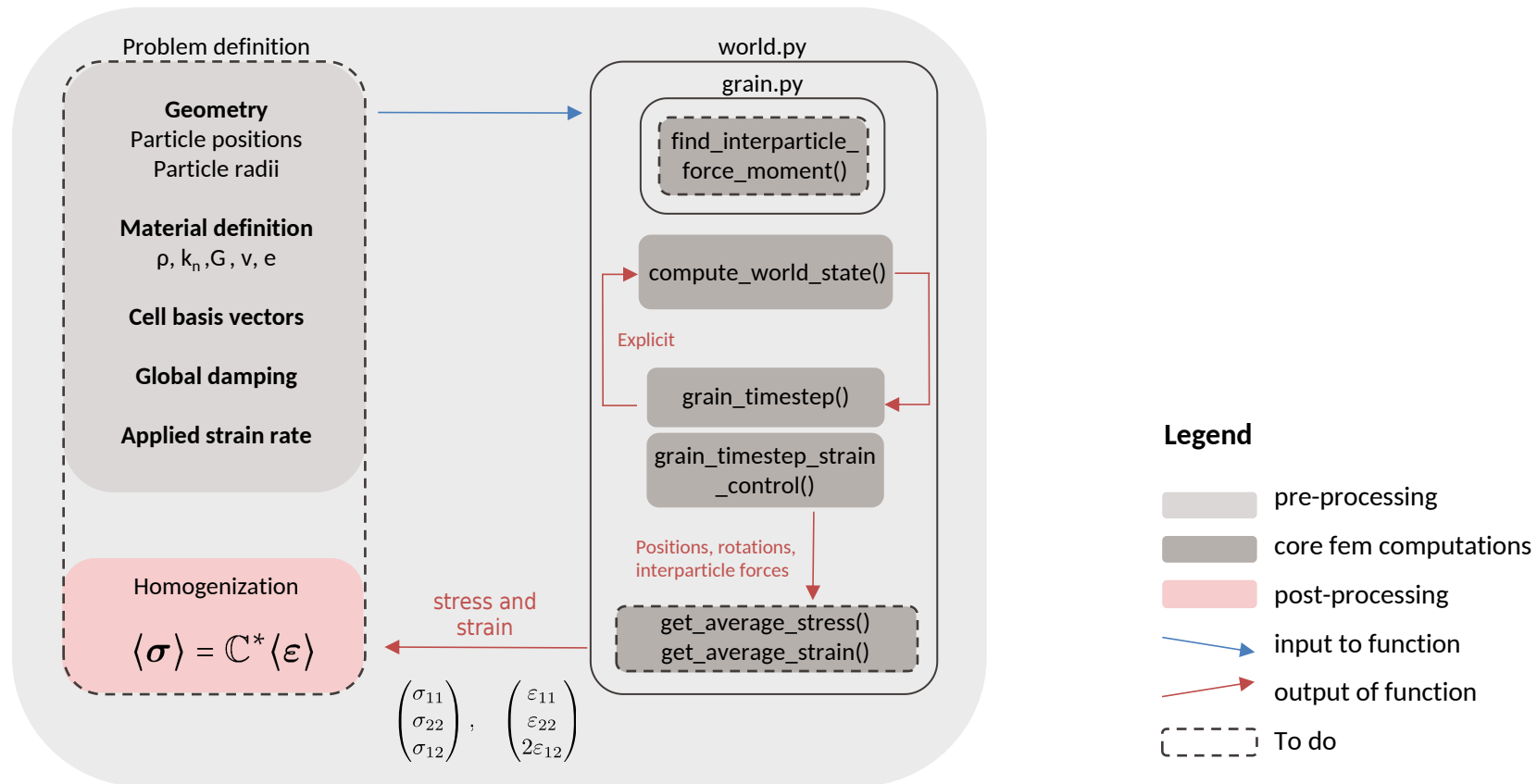
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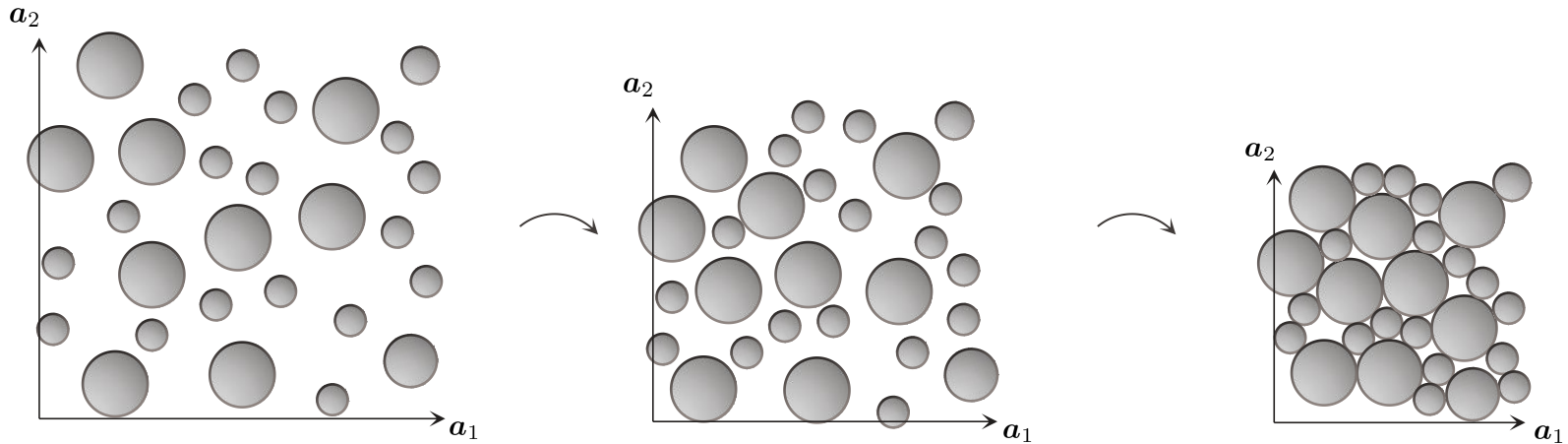
homogenized_stiffness_*.py



EPFL Exercise - Particle packing

We will compress the system from a gas state, and let it relax while imposing periodic boundary conditions.

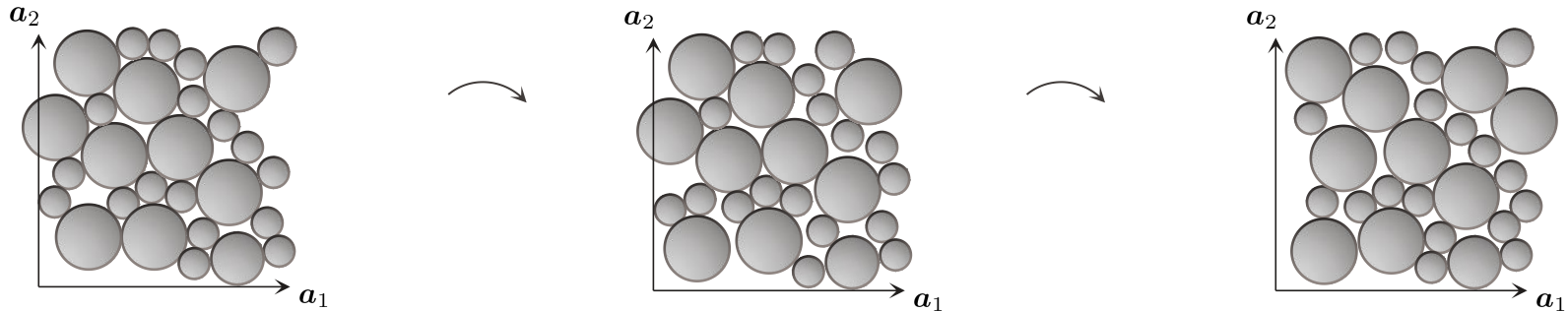
Phase 1: Compression



EPFL Exercise - Particle packing

We will compress the system from a gas state, and let it relax while imposing periodic boundary conditions.

Phase 2: Relaxation



Q: Why do we consider a **polydisperse** system?

EPFL Exercise - Particle packing

Particles:

$N = 225$

Size-bidisperse

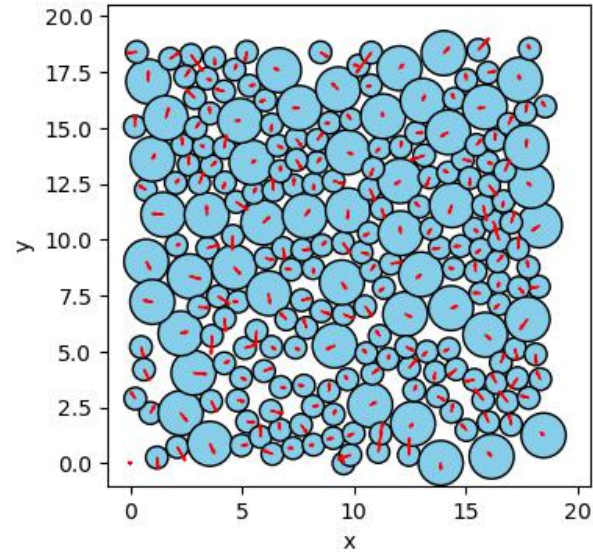
Objective:

- Create packing of
given packing fraction

$$\phi = \frac{\sum_p V_p}{V}$$

using:

- Periodic BCs $\mathbf{a}' = \mathbf{F}\mathbf{a} = (\mathbf{I} + \dot{\boldsymbol{\epsilon}} dt) \mathbf{a}$



Let's move to the Python notebook